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ABSTRACT

This report describes the planning, writing, evaluation, and results and recommendations of the Secondary School Mathematics Curriculum Improvement Study. This study was begun "to formulate and test a unified secondary school mathematics program (7-12) that will take capable students well into current collegiate mathematics" and to "determine the education required by teachers who will implement such a program." Also included in the report is a scheme for a taxonomy of objectives and course contents for each of the three courses developed (grades 7-8-9). Recommendations are that the program be reexamined for possible adoption as a curriculum for all junior high school students, that the preservice mathematics education of prospective junior high school teachers be reexamined, and that the unified approach developed for junior high school be extended throughout the senior high school. (FL)

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SECONDARY SCHOOL MATHEMATICS CURRICULUM IMPROVEMENT STUDY

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August, 1970

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SUMMARY

The Secondary School Mathematics Curriculum Improvement Study (SSMCIS) has two main objectives:

1) To formulate and test a unified secondary school mathematics program (7 - 12) that will take capable students well into current collegiate mathematics;

2) To determine the education required by teachers who will implement such a program.

To inaugurate the study, leading United States and European mathematicians and educators met in June 1966 to formulate a position paper stating the aims and procedures of the study, to construct a flow charted analysis of the proposed 7 - 12 mathematics courses, and to make detailed recommendations for the mathematical content of Course I. Using this detailed syllabus as a guide, a team of eight mathematics educators wrote a textbook for Course I during that summer. Each chapter was written by one writer, reviewed by the other writers and a consulting mathematician, and then revised for printing. Teachers' guides and solutions to exercises were written and distributed to the teachers.

In each subsequent year (1967, 1968) a two-week June working conference was held to review and revise the previous year's experimental text and to make specific recommendations for the content and teaching of a new course-Course II and Course III, respectively. As in the first year the writing team used the chapter guides developed by the June working conference to write the new texts for the following year.

Six junior high schools in the metropolitan New York area have participated in the experimental teaching of Courses I, II, and III from the initiation of the experiment. Each of these schools designated a team of two capable and interested teachers who taught all the pilot classes using the experimental textbooks. Each summer, while new materials were being written, six weeks of instruction was given to these teachers in preparation for teaching the new SSMCIS course. This instruction included 50 hours in the fundamental concepts underlying the unified mathematics program and 50 hours in contemporary methods of teaching those concepts.

The experimental teaching was evaluated in three ways. The director and project staff members made frequent visits to the classes for direct observation. The students were tested by examinations - prepared by the project staff - designed specifically to measure learning of important new concepts introduced in the courses. Teachers, staff, and consultants met at full day conferences to discuss progress and problems in the experimental teaching.

Results of the experimental teaching have shown that the new mathematics courses, based on fundamental concepts and structures, give promise of meeting the expectations of the proposed six year program.

Introduction

During the past decade the United States has been engaged in revising the elementary and secondary school mathematics curriculum - primarily by up-dating the existing traditional curriculum. Modest recommendations of the Commission on Mathematics have been largely accepted by curriculum and syllabus bodies and by writers of commercially produced textbooks. Implementation of this program by the SMSG has had wide acceptance and massive experimental use throughout the country.

Throughout all of our reform movements the traditional division of mathematics instruction into separate years of arithmetic, algebra, and geometry has been maintained. Beyond introduction of new concepts, little has been gained in bringing more advanced study into the high school through more efficient methods of organizing the subject matter. Bolder and more radical recommendations for the improvement of secondary school education in mathematics have been made both in this country, notably by the UICSM, and in Europe, notably in Belgium, Switzerland, and Denmark.

What has been called for is reconstruction of the entire curriculum from a global point of view - one which eliminates the barriers separating the several branches of mathematics and unifies the subject through its general concepts (sets, operations, mappings, and relations) and builds the fundamental structures of the number systems, algebra, and geometry (groups, rings, fields, and vector spaces). The efficiency gained by such organization should permit introduction into the high school program of much that was previously considered undergraduate mathematics.

In September 1965, the Commissioner of Education, Department of Health, Education, and Welfare, Office of Education, approved for support for a period of 18 months the Secondary School Mathematics Curriculum Improvement Study (SSMCIS), an experimental study whose objective would be the construction of a unified school mathematics curriculum for grades seven through twelve.

In June, 1967, continuation of this support was granted for an additional 36-month period ending June 30, 1970. This is a report of the activities and findings of the SSMCIS during this latter period, covering the writing and teaching of experimental Courses II and III, the revision and further, teaching of Courses I and II, and the final revision of Courses I and II.

Planning the 7 - 12 Program

In June 1966 a group of eighteen leading United States and European mathematicians and educators met for 20 days to outline the scope and sequence of a six year unified secondary school mathematics program. The first half of the conference was devoted to producing a complete flow charted analysis of the proposed course. Then topics planned for the seventh grade were expanded in working papers which outlined the mathematical content of each textbook chapter and made specific suggestions for writing and teaching these ideas.

Writing of Courses I, II, and III

During July and August 1966, a team of eight mathematical educators wrote the textbook for Course I, using the syllabus produced in June as a guide. Each textbook chapter was written by one writer, reproduced for review by the other writers and consulting mathematicians, and then rewritten, incorporating the reviewers' suggestions. Teachers' guides and solutions to exercises were written for each chapter. These notes, mimeographed and distributed to teachers of experimental classes, included discussions of fundamental mathematical ideas underlying each chapter, hints for possible class activity to accompany reading of the text, and suggested time allotment to the various topics. The Course I textbook was then published in three volumes.

To initiate the detailed planning for Course II and for the revisions of Course I, a pre-planning session was held on March 11-12, 1967. The recommendations of this pre-planning group were considered in detail at a working conference held in June, 1967, at which the full group of writers and consultants was present. This group produced a detailed set of writing guidelines, which were then used by the summer writing teams to produce the text materials for experimental Course II and to revise Course 1.

Beginning with a pre-planning meeting in December, 1967, and followed by a full working conference in June, 1968, a similar procedure was used to plan and write experimental Course III, to rewrite Course II, and to finalize Course I. In addition, in order to make the fullest possible use of the experience of two years of teaching Course I, ten of the experimental teachers re-wrote and expanded the original teachers' commentaries for Course I, which had been originally written by the authors of the various chapters. These chapter-by-chapter commentaries were then bound into a single volume and made available for public use along with the texts.

In 1969, the writing activities of the SSMCIS, supported by the Office of Education, consisted of rewriting Course III and making final revisions in Course II. Again, as in the previous year, a group of teachers of experimental classes worked during the summer, to rewrite the teachers' commentaries for Course II, which were then printed in a single volume for use in conjunction with the texts.

The texts and Teachers Commentaries for Courses I and II (1 each in two volumes) are now available for widespread classroom use, and are published and distributed by the Teachers College Press. When revision of Course III, now under way, is completed, the two volumes of the Course III text and the accompanying Teachers Commentary will be similarly available.

Education of Teachers

Each summer, beginning in 1966, the teachers of experimental classes have participated in a six-week program of special study at Teachers College, designed to prepare them to teach an experimental course in the following school year. This program of study was in two parts. The first was a course in the mathematical subject areas underlying the experimental course content, such as: abstract algebra, linear algebra, transformation geometry and probability. These subjects were taught from a modern and unified point of view. The second part of this program was a course in pedagogical methods of teaching secondary mathematics as a unified branch of knowledge, with special emphasis on the specific structures and principles to be covered in the following year of experimental teaching.

The following is a list of the teachers and the schools in which they taught the experimental classes (Course III) during the 1968-69 school year:

Elmont, New York
Schools: Alva T. Stanforth Junior High School
Sewanhaka High School
Teachers: Samuel Backer
Alexander Imre
Edward Keenan
Mary Murray

Leonia, New Jersey
School: Leonia High School
Teachers: Christine McGoey
David Swaim

New York, New York
School: Hunter College High School
Teachers: Douglas Bumby
Ruth Cohen
Richard Klutch

Teaneck, New Jersey
Schools: Benjamin Franklin Junior High School
Thomas Jefferson Junior High School
Teaneck High School
Teachers: Franklin Armour
Annabelle Cohen
Otto Krupp
Mary Renda

Westport, Connecticut
Schools: Bedford Junior High School
Coleytown Junior High School
Long Lots Junior High School
Staples High School
Teachers: David Fuys
Robert Keller
John Pepe
Daniel Sullivan

All teachers showed intense interest and cooperated splendidly in acquiring the spirit and content of the proposed new curriculum, and in teaching it. As a result of this training we now have a core of demonstration teachers and also a body of subject matter that must constitute teacher preparation in the future.

Teaching Courses I, II, and III

Six junior high schools in the New York Metropolitan area have taken experimental classes through Course I, II, and III. Five of these classes covered the text material in its first experimental revision. Another fifteen experimental classes have completed these three courses in the revised versions and another fifteen non-experimental classes in these schools have completed both Course I and II in their final revisions. Since the SSMCIS program is at present designed for those students in roughly the top 15% of their class with respect to mathematical ability, the original selection of students for the twenty experimental classes was made by the participating schools with prior mathematics achievement and scores on aptitude tests as main criteria.

Because the teachers of pilot classes were working as a team in the experimental class, they were often able to help each other with difficulties that arose in understanding or teaching the new material. Having had this year of team teaching experience, the teachers are now prepared to teach Courses I, II, and III on their own.

During the school year, the director and project staff members made frequent personal visits to observe the experimental teaching. Each class was observed at least four times. Visits to these schools included discussions with the teachers and administrators concerning progress and problems with the experimental course.

The teachers were further assisted by several full-day meetings at Teachers College where teaching problems were reviewed with selected consultants and the project director. At these meetings many teaching difficulties were resolved and valuable criticisms of the textbook were gathered.

Evaluation of Courses I, II, and III

The six year mathematics program introduces many new concepts into the secondary school mathematics curriculum and integrates both standard and new topics in a global organization not characteristic of existing programs. Student achievement in such a program cannot adequately be measured using conventional standardized tests. For this reason, student learning was tested by extramural examinations constructed by the project staff.

To guide construction of these and future measurement instruments, the textbooks were analyzed to produce a taxonomy of cognitive objectives. This taxonomy aided in delineating goals of instruction in terms of subject matter and related behaviors. The categories of behavior appear in Table I.

Each year, two examinations were prepared; one for a mid-year evaluation, and one for an evaluation at the end of the year. These instruments were used as a measure of the teachability as well as of the learning of the prepared content. They also guided the revisions that were subsequently made.

Although achievement on standardized traditional mathematics tests was not accepted as a measure of the success of the experimental program, it was of interest to determine at the start whether or not study in the experimental Course I affected learning of traditional topics. To accomplish this objective all students were administered the Sequential Test of Educational Progress - Mathematics, Form 3A - in September 1966 and again in September 1967. The test results clearly show that students in the project classes suffered no decline in mathematical skills when compared with students studying more traditional programs. Moreover, the achievement of these students on the project tests shows that they were learning to work with many new and powerful mathematical tools not a part of the traditional mathematics fare of seventh graders.

Future Activity

The Secondary School Mathematics Curriculum Improvement Study received support from the National Science Foundation in June 1969 to continue designing and experimenting with a unified secondary school program for college capable students in the senior high school. This support has enabled the project to produce Course IV in the six year sequence and to begin planning for Course V and VI.

Course IV, which had been written in the summer of 1969, was pilot-tested in five experimental schools during the 1969-1970 school year. As a result of this experimentation, Course IV will be revised during the summer of 1970 and will be available by the early Fall (1970). After further testing, the final version will be available by September 1971.

A tentative outline for Course V was developed during a two day conference of the advisory council in January 1970. This outline was expanded during a nine day conference of writers and consultants into detailed guides for the writers of the chapters in Course V. This course will be written during the summer of 1970 and experimented with in five schools during 1970.

The planning for and writing of Course VI will follow a similar procedure to that outlined for Course V with classroom testing during the 1971-72 school year.

By the end of the academic year 1973, the SSMCIS will have completed its task - to make a reconstruction of the secondary mathematics curriculum by presenting the subject as an integrated body of knowledge reflecting the spirit of contemporary mathematics.

Conclusions and Recommendations

After 5 years of classroom experimentations with students selected in the upper 15 to 20% of academic ability and taught by interested classroom teachers with special training in subject matter and pedagogy, a new curriculum in mathematics has been designed for the junior high school study, grades seven, eight, and nine. This curriculum breaks down the traditional barriers separating arithmetic, algebra, and geometry, and unifies the study through those fundamental concepts underlying all the branches, namely sets, relations, functions, and operations. The resulting curriculum is like a double helix in which the important structures - group, ring, field and vector space - form one strand, while the other strand consists of the important realizations: the number systems and the several geometries; synethetic, coordinate, vector, and transformation. Interwoven with both these strands are the activities and applications including the study of function, conditional sentences, statistics and probability.

Students who complete this three year program are advanced in knowledge more than one year beyond the present college preparatory program. This is accomplished by eliminating a great deal of traditional content that today is of little or no value in further study or application of mathematics. The increase in learning is also brought about by the unification of all the study under the more general concepts and structures of contemporary mathematics.

The teachability has been tested and verified, each course undergoing three years of thorough examinations and revision into its present form. Concomitantly, five doctoral studies researching the learning and teaching aspects of the program were completed. These studies were carried out by the research assistants associated with the project. They were:

Nicholas A. Branca - "Strategies in Learning Mathematical Structures," 1970.

James T. Fey - "Patterns of Verbal Communication in Mathematics Classes," 1968.

Michael J. Hoban - "Transformation Geometry in the Junior High School: An Evaluation of a Curricular Unit in the 7th Grade," 1970.

Stanley F. Taback - "The Child's Concept of Limit," 1969.

H. Laverne Thomas - "An Analysis of Stages in the Attainment of a Concept of Function," 1969.

A study in one school has shown that the same material can be learned by students of average ability, if pursued at a slower rate over a longer period of time.

It is strongly recommended that the program developed by SSMCIS be reexamined for adaptation as a curriculum for all students in the junior high school. It is also recommended that the SSMCIS program serve as a basis for re-examining the pre-service mathematics education of prospective junior high school teachers of mathematics. For these teachers, far more attention must be given to abstract and linear algebra, and to geometry of a modern variety, with less stress on advanced analysis. Lastly it is recommended that the unified approach be extended throughout the senior high school study to give college preparatory students an advanced knowledge of all aspects of mathematics on entering college and not only that of the calculus.

TABLE I

TAXONOMY OF OBJECTIVES

Mathematical Objectives

Structures:	Arithmetic and Algebra	Geometry	Probability	Analysis	
Fundamental Concepts	Sets	Operations	Relations	Mappings	Logic

Behavioral Objectives

I. Ability to recall definitions, notations, operations, concepts.

II. Ability to manipulate and calculate efficiently.

III. Ability to interpret symbolic data or processes.

IV. Ability to communicate mathematical ideas.

V. Ability to apply concept to a purely mathematical situation--solve problems.

VI. Ability to apply concept to problems in other situations--solve word problems.

VII. Ability to transfer learning to a new situation in mathematics.

VIII. Ability to construct or follow a mathematical argument.

Of course not all these categories apply to each subject matter topic, but the scale were checked against subject matter.

APPENDIX A

COURSE I CONTENT

Chapter

1 FINITE NUMBER SYSTEMS
Jane Anderson's Arithmetic
Clock Arithmetic
$(Z_{12},+)$ and $(W,+)$
Calendar Arithmetic
Open Sentences
New Clocks
Rotations
Subtraction in Clock Arithmetic
Multiplication in Clock Arithmetic
Comparison of $(W,\cdot)$ and Clock Multiplication
Division in Clock Arithmetic
Inverses in Clock Arithmetic
The Associative and Distributive Properties
Summary

2 SETS AND OPERATIONS
Ordered Pairs of Numbers and Assignments
What is an Operation?
Computations with Operations
Open Sentences
Properties of Operations
Cancellation Laws
Two Operational Systems
What is a Group?
Summary

3 MATHEMATICAL MAPPINGS
Assignments and Mappings
Mappings of Sets of Whole Numbers
Mappings of Clock Numbers
Sequences
Composition of Mappings
Inverse and Identity Mapping
Special Mappings of W to W
Summary

4 THE INTEGERS AND ADDITION
Introduction
Some New Numbers
The Integers and Opposites
Properties of $(Z,+)$
The Integers and Translations on a Line
Subtraction in $(Z,+)$
Subtraction as Addition of Opposites
Equations in $(Z,+)$
Cancellation Law

APPENDIX B

COURSE II CONTENT

Chapter

1 MATHEMATICAL LANGUAGE AND PROOF
Introduction
Mathematical Statements
Connectives: And, Or
Conditional and Bi-conditional Statements
Quantified Statements
Substitution Principle for Equality (SPE)
Inference
Direct Mathematical Proof
Indirect Mathematical Proof
Summary

2 GROUPS
Definition of a Group
A Non-Commutative Group
More on Permutations
Functional Notation
More Notation
Some Theorems About Groups
Isomorphism
Summary

3 AN INTRODUCTION TO AXIOMATIC AFFINE GEOMETRY
Preliminary Remarks
Axioms
Some Logical Consequences of the Axioms
A Non-Geometric Model of the Axioms
Other Models of the Axioms - Finite and Infinite
Equivalence Classes of Parallel Lines
Parallel Projection
Vectors - An Intuitive Introduction
Summary

4 FIELDS
What is a Field?
Getting Some Field Theorems Painlessly
Trouble with 0
Subtraction and Division in Fields
Fractions in Fields
Order in Fields
How Many Ordered Fields?
Equations and Inequations in $(Q,+,\cdot,<)$
Solving Quadratic Equations
Summary

Chapter

APPENDIX C

COURSE III CONTENT

Chapter

1 INTRODUCTION TO MATRICES
What is a Matrix
Using Matrices to Describe Complex Situations
Operations on Matrices
Matrices and Coded Messages
Matrices and Transformations
Transition Matrices
Summary

2 LINEAR EQUATIONS AND MATRICES
Linear Combinations of Equations
Solving Systems of Linear Equations
Solving Systems of Linear Equations, Continued
Homogeneous Linear Equations
Systems of Linear Equations and Matrices
Matrix Inversion
Summary

3 ALGEBRA OF MATRICES
The World of Matrices
Addition of Matrices
Multiplication by a Scalar
Multiplication of Matrices
Multiplicative Inverses in M_2
The Ring of 2 x 2 Matrices
A Field of 2 x 2 Matrices
Summary

4 GRAPHS AND FUNCTIONS
Conditions and Graphs
Regions of the Plane and Translations
Functions and Conditions
Functions and Solution of Equations
Operations on Functions and Asymptotes
Summary

5 COMBINATORICS
Introduction
Counting Principle and Permutations
The Power Set of a Set
Number of Subsets of a Given Size
The Binomial Theorem
Mathematical Induction
Summary