DOCUMENT RESUME

ED 077 720 SE 016 258

TITLE	Secondary School Mathematics Curriculum Improvement Study Information Bulletin 7.
INSTITUTION	Secondary School Mathematics Curriculum Improvement Study, New York, N.Y.
SPONS AGENCY	National Science Foundation, Washington, D.C.
PUB DATE	73
NOTE	34p.
AVAILABLE FROM	SSMCIS, Box 120, Teachers College, Columbia University, New York, New York 10027 (No charge while supplies last)
EDRS PRICE	MF-$0.65 HC-$3.29
DESCRIPTORS	*Curriculum; *Curriculum Development; Instruction; Mathematics Education; *Program Descriptions; *Secondary School Mathematics; Talented Students
IDENTIFIERS	*Secondary School Math Curriculum Improvement Study

ABSTRACT

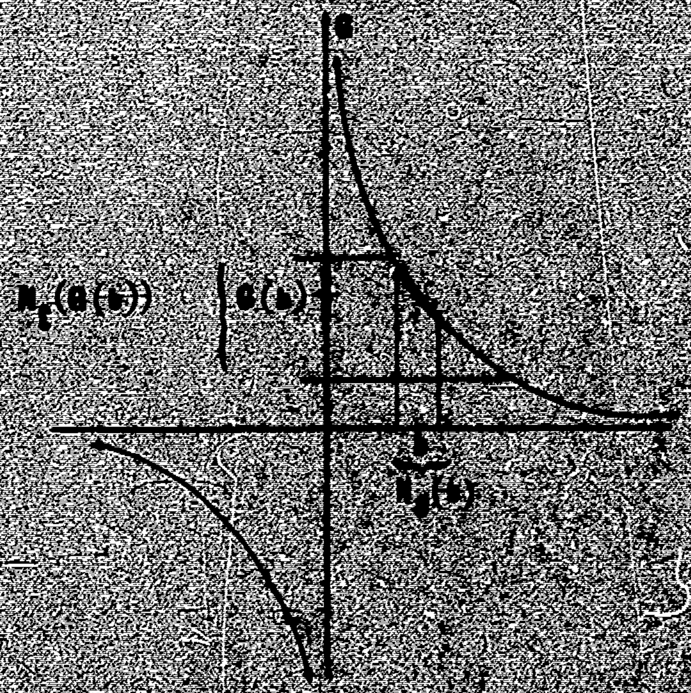
The background, objectives, and design of Secondary School Mathematics Curriculum Improvement Study (SSMCIS) are summarized. Details are given of the content of the text series, "Unified Modern Mathematics," in the areas of algebra, geometry, linear algebra, probability and statistics, analysis (calculus), logic, and computer programming. Evaluation activities, implementation, and teacher training are described. (DT)

ED 077720

Information Bulletin
Spring, 1973

7

SECONDARY SCHOOL MATHEMATICS CURRICULUM IMPROVEMENT STUDY



SUPPORTED BY THE
NATIONAL SCIENCE
FOUNDATION

SE 016 258

BULLETIN NO. 7 - SSMCIS

Preface

During the past 8 years, the Secondary School Mathematics Curriculum Improvement Study (SSMCIS) has been engaged in constructing a new curriculum in mathematical education under financial grants from the United States Office of Education and the National Science Foundation. The program developed is one that breaks down the barriers between arithmetic, algebra, geometry, trigonometry and the calculus. This is the final report to the nation at large on the program that has been developed and its applicability to various levels of student ability and interest and its availability to the profession at large.

A complete course of study for grades seven through twelve is now available. Starting with ten experimental schools and twenty classroom teachers involving approximately 300 students, the program has now been introduced to more than 20,000 students in more than 200 schools. The initial results show that the program is a challenging one and students are enthusiastic about the material and its presentation. Large scale innovations have been made in the state of Utah, and in Los Angeles, Philadelphia, and the New York Metropolitan Area.

Since there is a definite need for secondary school teachers and supervisors to be knowledgeable of this program for future planning of their own school program, we present a comprehensive exposition of our Unified Modern Mathematics. This includes a description of the design, development, and substance of the program, a description of teacher preparation, student selection and evaluation--further studies that will be completed during the coming year, a list of collaborators, and information on the source and availability of all materials for the program.

We are indeed proud to present this program to the mathematics education community of the United States of America.

Howard F. Fehr
Director

TABLE OF CONTENTS

Issued May 15, 1973 Teachers College, Columbia University, New York

THE SECONDARY SCHOOL MATHEMATICS CURRICULUM IMPROVEMENT STUDY

DESIGN AND SUBSTANCE

The Classical Viewpoint.

Traditional mathematics grew out of the need for understanding the physical environment and out of this practical knowledge there was created an idealization of physical space (Euclidean geometry) and a system for counting and measuring (whole and rational numbers). As commerce, navigation, and exploration grew, the need for further mathematics led to higher specializations called algebra and analysis. Almost 300 years ago all this knowledge was organized into four branches of mathematics--arithmetic, algebra, geometry, and analysis--each considered a closed and separate field of investigation. This organization became the manner of presenting mathematics as a subject for school instruction, that has presisted for nearly 200 years.

The Contemporary Viewpoint.

Mathematics, as a branch of knowledge, no longer holds to this classical point of view. From the developments in understanding number, organizing algebraic systems, and creating new geometric spaces, along with the emergence of set theory, formalism, and the concept of structure, a new contemporary viewpoint of mathematics has come into being. As early as the 1930's mathematicians recognized that certain fundamental concepts underpinned all the branches of mathematics, and that structural concepts gave possibilities for organizing all mathematics into a unified body of knowledge. As a consequence, contemporary mathematics uses the basic notions of sets, relations and mappings, the algebraic structures of group, ring, field, and vector space; and the topological structures of metric and linear space, compact space, normed and vector space. This structuring permits greater understanding, efficiency in learning, and uncovers concepts, and theories previously hidden by the traditional separation.

It is this contemporary viewpoint of mathematics that shows how to restructure our secondary school mathematics program.

Today, the United States of America is the only highly developed country in the world that still adheres to the traditional mathematics study separated into (a) a first year of algebra and only algebra (b) followed by a year of geometry and only geometry at the end of which the study is dropped forever (c) then another year of algebra, much of it repeated from the first course which is presumed to have been forgotten. Except for highly mathematically inclined students this is the total mathematical learning of high school graduates. It is an out of date program, mostly useless for genuine application and reflects a 1901 point of view.

The Goals of School Mathematics Instruction.

The mathematics we teach to our students today should be relevant to their needs in the society of tomorrow in which they will live. Thus the mathematics we teach should reflect the manner in which the subject is conceived for future use. To this end we must first of all be concerned with the formation of the intellect--the ability to do cognitive thinking. What we teach must develop the human mind in its capacity to understand and interpret numerical, spatial, and logical situations occuring in the physical universe and life within it, and to approach problems with a scientific, questioning, and analytic attitude. All of our students must come to know mathematics as it is conceived today and what material it deals with, what types of thinking (not only axiomatic) it uses, what it accomplishes, and how it is invading almost all other domains of human activity. This is the primary purpose of the SSMCIS program.

Our instruction must have an "informational and skill" dimension also, inasmuch as it is charged with transmitting from one generation to the next, that inherited knowledge and skill to use it which is considered basic and useful in the years ahead. This information should be acquired during the process of developing mathematical thinking. This target of instruction permits us to drop a great deal of the traditional content no longer considered useful and to choose more general and more unifying concepts as basic.

Further, it is the usefulness of our subject that has maintained it as a main discipline of educational endeavor. Our instruction serves to develop the capacity of the human mind for observation, selection, generalization, abstraction, construction of models and procedures for use in solving problems in the other disciplines. Unless the study of mathematics can operate to clarify and to solve human problems it has indeed only narrow value.

The Design of Experimentation.

To initiate the development of a curriculum to meet the foregoing goals, in 1965 a group of distinguished mathematicians and mathematical educators from Europe and America were convened to formulate a scope and sequence for a program in mathematics for capable students in grades seven to twelve. (See Appendix, page 12) The program is based on the ideas of sets, relations. mappings and operations. On these fundamental concepts are built the structures, group, ring, field, and vector space. The realizations of these structures--the number systems and the several geometries and all the activities that take place in their study form the important concepts and uses of mathematics. This unified organization not only permits the calculus to be studied in the 11th and 12th school years but it also provides the study of genuine modern applications of probability, statistics, computer oriented problems, linear programming, numerical analysis as well as the usual application of calculus to mechanics and kinematics.

Each year of experimental study was preceded by a summer in-service institute where 20 teachers were trained to understand the mathematics and how to teach it in order to secure the desired goals. During the first year of each course two-teacher teams taught each experimental class alternately, one teaching and the other observing student and teacher reaction. At intervals these teachers reported back to the SSMCIS staff consultants on the results of the teaching. During the following summer the course was revised and rewritten to take into account the criticisms and suggestions of the teachers. After a second revision, the course was put into final form and released to the public. Thus, each course has essentially undergone three years of tryout before achieving its present form. The outcomes were tested by both cognitive and affective domain instruments defined by the general goals of the program.

The Content of the SSMCIS Unified Mathematics

The algebra study begins with finite arithmetics as numbers on a clock, and develops the fundamental notions of operation, commutativity, associativity, identity element, and inverse element.

Besides the traditional binary operations of addition, multiplication, powers, and their inverses, other operations are examined, for example those of max, min, $[(a,b) \longrightarrow a^2 + b^2]$, lcm, gcf, midpoint, tri-point, composition, and many others. At all times sets of numbers, points, or elements that constitute a group are singled out. In turn the number systems--whole numbers, integers, rationals, reals, and complex, are introduced semiformally and related to the associated structure--group, ring, field. Matrices are introduced in grade 9 with illustrations of their many uses, for instance as transformation operators on points in two and three space.

After mappings of various sets have been studied (numbers on a line, elements in a plane, etc.), functions are developed on each of the several number systems. Operations on functions, and graphs of functions are examined. Special functions are singled out for extensive treatment, for example, the polynomials (quadratics in particular), rational functions, trigonometric functions, logarithmic and exponential functions, the identity and inverse functions. All this algebra comes into extensive use in the development of the calculus.

The geometry program is a prominent and important one in SSMCIS, in which however, there is none of the usual treatment of synthetic Euclidean geometry. There is a review of the common line figures in a plane, measurement of line segments and angles, coordinatization of a line, and lattice points in a plane. By the use of drawings, paper folding, mirrors, and other physical devices, transformations of the plane are introduced as mappings. Reflections in a line and a point, translations, rotations (half turns), and glide reflections, with their distance preserving property lead to the group of isometries. In turn these transformations are used to develop many of the properties of plane Euclidean geometry. Dilations are also introduced and related to similarity. Subsequently the transformations are studies in a coordinatized plane and related to 2 x 2 matrix operators.

A synthetic axiomatic affine plane geometry, with three axioms and several definitions is developed in grade 8, from which 15 to 20 theorems are proved. Finite models (4 points, 9 points, committee structure) are given and the theorems interpreted in these models. This geometry is subsequently extended to the coordinatized affine plane, and informally to 3-space affine geometry. There are units of study on measures of areas and volumes of common geometric regions, and an informal study of 3-dimensional Euclidean space. Our students study space by transformations, by coordinates, by synthetic methods and by vectors. The latter enters through the linear algebra program which is new to secondary school mathematics.

This study of linear algebra begins with the algebra of matrices, the solution of systems of linear equations by the Gauss-Jordan method which is then adapted to a tableau algorithm. Systems are expressed in matrix notation. Later, vectors are introduced as ordered pairs, ordered triples, or generally ordered n-tuples of real numbers. Addition and scalar multiplication of vectors leads to the geometry of affine lines and vector lines in a plane, and then of lines and planes in 3-space (or n-space). This study culminates in the definition of an abstract vector space structure which is exhibited by many models of earlier study. The study of sub-spaces completes this first approach. A graphical interpretation is given to the solution of linear programming problems.

The subsequent study returns to linear equations represented in parametic form, the elimination of parameters to obtain the standard form and vice-versa. Then linear sums or linear combinations, the generation of spaces (spanning), linear dependence, basis and dimension are examined. In the entire study, an associated geometric affine vector-space serves for illustration. With the introduction of the inner product, orthogonality is defined, the norm and distance functions are developed, and the Euclidean vector space of two and three dimensions is achieved. A chapter in the eleventh school year develops linear mappings and the concepts of kernel and range of linear mappings. The study concludes with linear programming using the simplex method.

Probability and Statistics appear as chapters of instruction in every grade, seven through twelve. The study begins with an a-posteriori approach by recording the relative frequency occurrence of outcomes in experiments, and the assignment of probability measure. With dice, spinners, coins, etc. an a-priori probability assignment is made on the expected uniformity of outcomes. The collection of numerical data and its graphical representation by histograms and frequency polygons leads to measure of central tendency--mode, median, arithmetic mean, and to variance--range and standard deviation. Here Σ notation and its use is introduced. In the ninth year, probability is approached in a formal manner with the axioms of a probability field--the outcome set, the power set, the probability space and the probability measure. Events are related to subsets of the outcome set (the power set) and their intersections. Combinatorics (permutation and combinations) are treated sufficiently to provide material for probability problems.

In the tenth year the study is extended to conditional probability, from which arise the concepts of dependent and independent events, Bayes' theorem, random variables and mathematical expectation. In the eleventh and twelfth years the study of probability includes many genuine applications of the subject--simulation, Bernoulli experiments, the binomial distribution, and Markov chains. With the calculus, the theory is expanded to include infinite probability space.

Analysis (the calculus) begins in the eleventh year with a contemporary approach to continuity via the metric topology of a line and a plane. By the use of the topological concepts of neighborhoods, continuity at a point and over an interval is developed intuitively and then formalized. This theory is then used to define limits (at a point) without recourse to sequences. The derivative of functions is approached by linear approximations to the graphs of the functions, followed by the usual development of theory, techniques, and applications of differentiation. The integral calculus is introduced through summations of rectangular areas utilizing piece-wise step function. The analysis program thus has a contemporary approach and covers most of the material demanded by the CEEB Advanced Placement A-B Examination, (although this is not an aim of our program).

For a brief period in the [illegible] logic is studied as a separate entity. Here the study treats quantifiers $\forall$ and $\exists$, connectives 'and' and 'or', the negation "not", implications, bi-implications, their symbols and their relations through the media of truth tables. The unit closes with an explanation of inference schemes (and a hint of the algebra of logic). This study is not used for the purpose of formalizing all subsequent study, but rather as a means for verifying theorems which are proved upon given conditions.

The computer program starts with flow-charting in the seventh grade. At the start of the tenth grade BASIC is taught, programs for solving numerical problems are written, and finally they are given to the computer through relayed consoles. Thereafter every mathematical topic subject to numerical computation procedures includes problems for programming and computer solution.

A feature of the twelfth year program is a set of booklets for individualized teacher-student instruction, according to the interest of the student. Each booklet stresses the application of the unified mathematics previously learned and is intended for one or two months of study. The five booklets are on Statistical Inference, Linear Algebra, Algebraic Structures, Differential Equations and Geometric Transformations.

Summary

Throughout the western world, a trend fast gaining full recognition is the restructuring of classical mathematical education into a contemporary unified setting. This is the goal of SSMCIS. At the end of six years of study it will provide both skills and concepts in

(1) A contemporary viewpoint of algebra as a study of structures, their number system realizations and the activities derived therefrom. It will prepare college bound students to begin rigorous abstract algebra and linear algebra study at the collegiate level.

(2) A modern viewpoint of geometry as a study of spaces eventually related to algebraic structures, especially those of vector spaces and linear algebra. This must be common knowledge of all future educated laymen.

(3) An approach to the study of all mathematics via the concepts of sets, relations, functions, operations, and structures binding the entire instruction, by a spiral approach to learning, into a unified body of knowledge.

(4) Application of mathematics, not only to physics, but to new areas in the behavioral sciences where probability, finite mathematics, and statistics are important.

(5) An intuitive non-rigorous, but correct and precise introduction to analysis via continuity and elementary metric topology, including one variable calculus.

The modification of this six year program, by including more applications, by offering more opportunity to solve problems, by concretization of the more abstract concepts and structures, and by eliminating those parts that can be studied after leaving high school, can form the core program--the common mathematical knowledge for a majority of secondary school students.

SSMCIS EXPERIMENTAL SCHOOLS AND TEACHERS

The development of mathematics curriculum, like the SSMCIS program, can not go forward without the commitment and cooperation of the classroom teacher. From the very first try-out of the experimental version of Course I in 1965, the SSMCIS teachers have provided their students in experimental classes with sound and enthusiastic teaching. Unable to acknowledge all of the many teachers who played a part in the experimental teaching of the SSMCIS materials, we cite the five school systems which have been involved with experimental teaching of Courses 1-6 over the past eight years.

Elmont Public Schools, Elmont, New York
Hunter College High School, New York, New York
Leonia Public Schools, Leonia, New Jersey
Teaneck Public Schools, Teaneck, New Jersey
Westport Public Schools, Westport, Connecticut

TEXTBOOK OUTLINE

To give a birdseye view of the scope and sequence of the SSMCIS program, the next page outlines the textbook material by course and chapters for all six years.

	Chapter	Title	Chapter	Title
COURSE 1	1	Finite Number Systems	8	Sets and Relations
	2	Sets and Operations	9	Transformations of the Plane
	3	Mathematical Mappings	10	Segments, Angles, Isometries
	4	Integers and Addition	11	Elementary Number Theory
	5	Probability and Statistics	12	The Rational Numbers
	6	Multiplication of Integers	13	Some Applications of the Rational Numbers
	7	Lattice Points in a Plane	14	Algorithms and Their Graphs
COURSE 2	1	Mathematical Language and Proof	7	Real Functions
	2	Groups	8	Descriptive Statistics
	3	An Introduction to Axiomatic Affine Geometry	9	Transformations of the Plane, Isometries
	4	Fields	10	Length, Area, and Volume
	5	The Real Number System	Appendix A	Mass Points
	6	Coordinate Geometry		
COURSE 3	1	Introduction to Matrices	6	Probability
	2	Linear Equations and Matrices	7	Polynomials and Rational Functions
	3	Algebra of Matrices	8	Circular Functions I
	4	Graphs and Functions	9	Informal Space Geometry
	5	Combinatorics		
COURSE 4	1	Programming in BASIC	6	Linear Programming
	2	Quadratic Equations and Complex Numbers	7	Sequences and Series
	3	Circular Functions, II	8	Exponential and Logarithmic Functions
	4	Probability: Conditional Probability and Random Variables	9	Vector Spaces and Subspaces
	5	Algebra of Vectors		
COURSE 5	1	Introduction to Continuity	6	Further Study of the Derivative
	2	More About Continuity	7	Linear Mappings and Linear Programming
	3	Limits		
	4	Linear Approximations and Derivatives	8	Probability: Expectation and Markov Chains
	5	Properties of Derivatives	9	Integration
COURSE 6	1	Infinity		**BOOKLETS**
	2	Conics	A.	Introduction to Statistical Inference
	3	Circular Functions-Analytic Properties	B.	Determinants, Matrices and Eigenvalues
	4	Exponential and Logarithmic Functions--Analytic Properties	C.	Algebraic Structures, Extensions and Homomorphisms
	5	Integration Techniques and Applications	D.	An Introduction to Differential Equations
	6	Probability: Infinite Outcome	E.	Geometry Mappings and Transformations

REPORT ON SSMCIS EVALUATION ACTIVITIES

This report covers the SSMCIS evaluation activities from Fall 1971 to Spring 1973. Owing to the limitations of space, many details have been omitted. These details and additional information can be found in the various SSMCIS Evaluation Reports discussed below.

Cumulative Progress Test

Bulletin No. 6 contained a discussion of a special examination given to ninth graders completing Course 3 in the Spring of 1971. A report of the results of that administration is given in Technical Report 6. Items from the three forms of the special examination were selected that were particularly appropriate for the evaluation of performance in the first three courses. (Recall that many of the items in the special examination were used for comparision with the performance of other groups and not necessarily to measure achievement in SSMCIS courses.) The selected items, plus additional items written to fill out a topic-by-course matrix designed to assure comprehensive coverage of the courses, were incorporated into a Cumulative Progress Test.

The Cumulative Progress Test was first administered in the spring of 1972. It was taken by 9 classes of students in schools that had participated in the development of the SSMCIS materials and by 23 classes of students in other schools. These classes provided normative data for the test, which will soon be available, along with a manual, for use as a comprehensive achievement test for SSMCIS Courses 1 to 3.

Student Opinion Survey

The original Student Opinion Survey, described in Bulletin No. 6, has been modified for readministration in SSMCIS and non-SSMCIS classes. Two forms have been prepared, both in an agree-disagree format. Both forms contain a set of items on the student's opinion of his mathematics course and his mathematics textbook. Form A contains, in addition, a set of items dealing with attitudes toward mathematics in general, the difficulty of learning mathematics in general, the difficulty of learning mathematics, and the place of mathematics in society; these are from Aiken's Revised Math Attitude scale and two scales used in the International Study of Achievement in Mathematics. A third scale from the International Study, one dealing with attitudes toward mathematics as a process, is contained in Form B, along with a set of specially written items dealing with descriptions of the student's mathematics class and his attitudes toward mathematics as a creative activity, the need to give reasons in mathematics, the unity of mathematices and the usefulness of mathematics. To increase the information obtainable in a single class meeting, the two forms will be distributed more or less at random within a class, and each student will take only one form. Since information is not necessary on each student, but only on each class, it is not necessary that every student respond to every item.

The scales will be administered to students in SSMCIS and comparable non-SSMCIS classes in several Maryland counties in the spring of 1973. Data from this administration will be used to estimate the effects of the SSMCIS program on students' attitudes. In addition, the two scales will be given at the same time to classes of SSMCIS students from schools participating in the development of the SSMCIS materials. Students in these experimental classes took the earlier Student Opinion Survey in 1970 and 1971, so some measure of stability and change in SSMCIS students' attitudes will be obtained.

Regents Examinations

Special examinations in lieu of the Tenth Year Mathematics and Eleventh Year Mathematics Regents examinations for June 1972 were written for Course 4 and Course 5 students in New York State by the SSMCIS staff in collaboration with SSMCIS teachers and representatives from the New York State Bureau of Mathematics Education.

As of December 1972, and until further notice by the State Education Department, New York State schools offering SSMCIS courses in the tenth of eleventh grade may grant Regents credit by certification based on local final examinations. This certified credit is to be made by the school principal and does not involve a numerical Regents grade. Further information regarding this ruling, together with sample examination questions from the special examinations previously approved for use with SSMCIS students, may be obtained from the SSMCIS office.

College Board Examinations

A first investigation of SSMCIS students' performance on College Entrance Examination Board tests was undertaken in 1971 in cooperation with the Mathematics Department of the Educational Testing Service (ETS). Several classes of SSMCIS students took the Mathematics Section of the Preliminary Scholastic Aptitude Test (PSAT) as part of the regular administration in October 1971. In addition, several other classes took a pretest form of the Level II Mathematics Achievement Test as part of the pretesting program in Spring, 1971.

Although the SSMCIS students were a select group, in comparison with other students who took the tests, some comparisons were possible. The data, although limited, suggested no serious bias in the examinations either for or against SSMCIS students. It appears that the SSMCIS students' relative ignorance of traditional Euclidean geometry is more than compensated for by their superior knowledge of other topics, such as functions. More information on the first study is available in Technical Report 4, which contains a memorandum prepared by members of the ETS Mathematics Department.

A second, more carefully controlled study of SSMCIS students' performance on the PSAT was begun this year with the identification of a larger sample of SSMCIS students who took the PSAT in October 1972. These students' performance will be compared with the performance of students of comparable verbal ability who were not in SSMCIS classes. The data from the second study should provide more exact and more detailed information on the relative advantage or disadvantage SSMCIS students might have on one of the major examinations used to predict success in college.

Meanwhile, informed reports to their former teachers by SSMCIS students now in college indicate that they are well prepared not only for their mathematics courses but also for those courses in other fields that use such mathematical topics as probability, statistic, and linear algebra. SSMCIS students report that they have an overview of mathematics that enables them to see the relevance and relatedness of certain topics in ways not always available to their contemporries. Such reports are encouraging, but more SSMCIS students will have to complete the program before a more definitive assessment can be made.

UNFINISHED BUSINESS FOR 1973-1974

In light of the program accomplishents made by June 30, 1973, the following activities will help in the attainment of ultimate objectives of the SSMCIS project.

A. The maintenance of the SSMCIS project office for information and communication.

(1) To distribute information on the program (Bulletins, availability of teaching materials, etc.) to teachers, supervisors, school districts, and teacher training personnel.

(2) To give information and counsel on available manpower and materials for summer and academic year institutes, workshops, leadership conferences, regional in-service training courses and undergraduate training programs.

(3) To oversee the publication by Teachers College Press and commercial publishers of SSMCIS materials so that

these materials are available at all times in the immediate future. This includes possible cooperation with any dissemination and innovation of SSMCIS Unified Mathematics by other institutions, for example, the National Institute of Education.

B. The completion of the evaluation program.

(1) To continue liaison with Educational Testing Service and College Entrance Examination Board on the achievement test results of SSMCIS students.

(2) To finalize comprehensive test for (i) Courses 1 through 3, and (ii) Course 4 and 5.

(3) To analyze the results of attitude tests administered to SSMCIS students the last few years and to be administered to SSMCIS and comparable non-SSMCIS control groups in Spring 1973.

C. The development of guidelines for student transfer.

(1) Transition from the tranditional program to SSMCIS. Two major problems confront many school systems considering the adoption of the SSMCIS program. They are the four year high school and placement of students entering the program at the 8th or 9th school year.

Since SSMCIS is a six year secondary school program designed for the upper 15% to 20% of academic ability, it is normal to begin the program in the seventh grade. Difficulties in instituting this plan have led to the following alternative proposals.

(a) Begin the program in grade 8 with Course 1 and proceed as far as possible in the next five years.

It should be possible to study the first five courses thoroughly, enabling a student to go far beyond the traditional secondary mathematics program.

(b) If a student enters Course 2 in grade 8 without previous study necessary topics in Course 1 can be studied concomitantly with Course 2.

(c) Urge, whenever qualified teachers can be found, that in 8-4 systems the program be introduced for able students at the start of the seventh grade.

(d) If extremely capable students, with no previous knowledge of the program, enter in grade 9, study should begin with Course 2 supplemented by essential topics from Course 1. It should be possible to study the first five courses if the program is carefully and intelligently accelerated.

During the ensuing year it is essential that these alternatives be defined completely so that a complete picture may be given. Specific suggestions as to topics necessary for supplemental study and ways to accomplish acceleration must be made.

2. Transition from SSMCIS to traditional program.

Students who complete one or more years of study of SSMCIS can gain a year of study in the traditional program. In general, a student finishing Course 1 should enter first year traditional algebra at the eighth school year in the Advanced Placement Program. A student finishing Course 2 at the end of the eighth grade should enter the senior high school tenth year geometry program. A student moving from the end of Course 3 should enter the second year of algebra in the Advanced Placement Program, doing a little vagabonding in the geometry course on circle properties and some construction problems.

Here again, it is essential to spell out these suggestions completely so that logical choices can be made.

D. Supplemental drill material on fundamental skills.

A common concern expressed by teachers and the public at large relates to mastery of fundamental skills. Many teachers seem to feel that contemporary mathematics programs neglect mastery. Still one must agree that there is a vast difference in true understanding and rote learning. Mastery is essential as one proceeds from one stage to another in the learning process. If mastery is combined with understanding, economy in the learning process is produced. Capitalizing on this economy and the greater efficiency of organization, the scope of the SSMCIS program is much broader than a traditional program. The SSMCIS project will continue to focus on this problem in the coming year. It will look at the problem of mastery of fundamental skills in greater depth in an attempt to provide sufficiently detailed guidelines of options for supplemental study so that additional emphasis may be given to fundamental skills if weaknesses appear.

UNIFIED MODERN MATHEMATICS, COURSES 1-6 were prepared by the Secondary School Mathematics Curriculum Improvement Study with the cooperation of

Nicholas Branca, Stanford University
John Camp, Wayne State University
Gustave Choquet, Universite de Paris, France
Ray Cleveland, University of Calgary, Canada
John Downes, Emory University
Howard F. Fehr, Teachers College, Columbia University
James Fey, University of Maryland
David Fuys, Teachers College, Columbia University
Allan Gewirtz, City University of New York
Abraham Glicksman, Bronx High School of Science, New York
Richard Good, University of Maryland
Vincent Haag, Franklin and Marshall College
Thomas Hill, University of Oklahoma
Julius Hlavaty, National Council of Teachers of Mathematics
Michael Hoban CFC, Iona College, New York
Meyer Jordan, City University of New York
Howard Kellogg, Brooklyn College, Brooklyn, N. Y.
Jeremy Kilpatrick, Teachers College, Columbia University
Harold Leonard, State University College at Oneonta, New York
Norbert Lerner, Cortland State College, New York
Howard Levi, City University of New York
Edgar R. Lorch, Columbia University
Richard C. Pocock, Houghton College, New York
Lennart Råde, Chalmers Institute of Technology, Sweden
Andre Revuz, University de Paris, France, Columbia University
Harry Ruderman, Hunter College High School, New York
Alice Schafer, Wellesley College
Willy Servais, Athene, Morlanwelz, Belgium
Harry Sitomer, C. W. Post College
Philip Smith, Southern Connecticut State College
Hans-Georg Steiner, University of Karlsruhe, Germany
Marshall H. Stone, University of Massachusetts
Stanley Taback, New York University
H. Laverne Thomas, State University College at Oneonta
Albert W. Tucker, Princeton University
Bruce Vogeli, Teachers College, Columbia University
Lucian Wernick, Teachers College, Columbia University
Paul White, University of Southern California
Joseph Zahn, Teachers College, Columbia University

SELECTION OF STUDENTS AND SSMCIS IMPLEMENTATION

From a school administrative point of view, the adoption of an accelerated program of study, such as SSMCIS, largely depends upon the availability of sufficient numbers of capable students and qualified teachers. School personnel responsible for the selection and placement of SSMCIS students are confronted with the task of identifying sixth grade students who would most likely profit from their study of SSMCIS.

A 1972 survey of 28 schools using the SSMCIS program, indicated that the following types of selection criteria were most frequently employed as a basis for SSMCIS student selection:

(1) sixth grade teacher's recommendation,

(2) intelligence test scores,

(3) sixth grade mathematics grade, and

(4) mathematics achievement test scores.

In most schools, some combination (usually (1) and (2)) of these criteria was used in selection.

The selection procedures described by the 28 schools can be grouped into four types of selection strategies.

(1) Traditional. Selection procedures, tests, and number of students selected are similiar to those for the school's traditional accelerated mathematics program.

(2) Highly selective. Initially, students are selectively chosen for SSMCIS Course 1, usually on the basis of IQ and achievement test data (upper 10-15%). After a few years, when Course 1 and 2 teachers have had a chance to gain experience with the materials, a larger proportion oi' seventh and eighth graders are included in SSMCIS classes.

(3) Less selective. Wishing to minimize the possibility of excluding potentially successful students from the SSMCIS program, a school selects a large number of students to study Course 1. Students who are not successful in SSMCIS transfer to a traditional mathematics, usually at the end of grade 7 or 8.

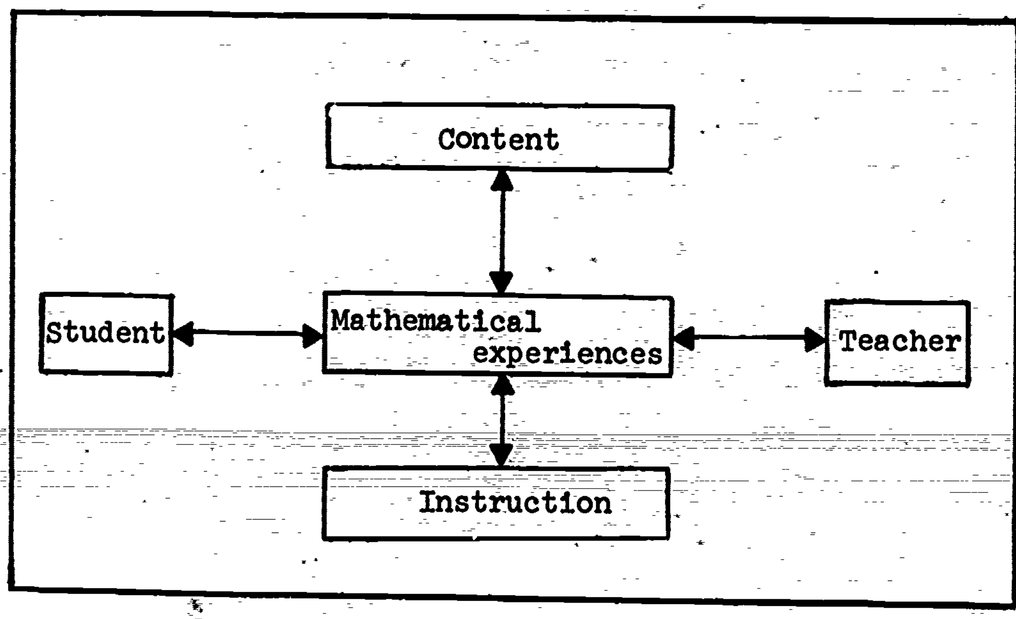
(4) Try-out. Based upon student performance in Course 1 during the first 4-10 weeks of school, a group of about 60 students is split into a SSMCIS class and a somewhat accelerated but traditional mathematics class.

At present it appears that schools initially adopting strategy (2) and then (3) develop the most successful implementation of the SSMCIS materials. Once teachers have had the opportunity to teach the Course 1 and 2 materials, their experience and expertise with these materials makes possible the inclusion of a wider ability range of students in SSMCIS, especially in Course 1.

The absence of specific guidelines for the selection of SSMCIS students necessitates careful evaluation of student performance by teachers of Courses 1 and 2. Early identification of students who have difficulty with the material can be coupled with special instruction and assistance for these students from the very beginning of their study. Teachers in SSMCIS experimental classes have reported several instances where students have struggled through Course 1 or Course 2 and then blossomed in later SSMCIS study. However, encouragement and individual assistance from the teacher are required.

SSMCIS AND TEACHER TRAINING

Often, curriculum reform in school mathematics is equated with changes in mathematical content--for example, the introduction of new topics or the presentation of a new approach to a topic. However, curricular change is a complex and dynamic process, involving components other than content. The diagram below depicts mathematics curriculum as a system of four interacting components: (a) the content, (b) the student, (c) the teacher, (d) instruction.



As a curriculum evolves, the various components undergo change--some more rapid than others. While the major thrust of SSMCIS has been to effect a change in the content component, there has been extensive involvement in the training of future SSMCIS teachers. Both pre-service and in-service training of teachers, especially at the junior high school level, are important for the sound and productive implementation of the SSMCIS program.

Background for Courses 1 - 3.

The textbooks of the SSMCIS program indicate the mathematics study of knowledge that is pre-requisite to the successful teaching of the material. Fundamentally the mathematical structure at the secondary school level is a miniature model of study at the collegiate and higher levels of study. Junior high school teachers should have a firm understanding of all the mathematics they will teach in Courses I, II, and III. This understanding should be at a high cognitive level.

It is easier to list courses that teachers should have had, rather than subject matter, because this is the way the content is usually presented in pre-service teacher education programs. All the courses which follow can be thought of as undergraduate. These courses, while mathematically oriented, should contain pedagogy. The intention is to prepare high school teachers of mathematics, not research mathematicians. However, any person who has had a recent undergraduate major in mathematics will have certainly me\ ~ ~t of this subject matter background requirement.

1. A course in foundations; set theory, logic, relations and mappings. 3 s.h. (3 semester hours)
2. A course in abstract algebra; groups, rings, fields, morphisms, vector spaces. 3 s.h.
3. A course in matrices and linear algebra. 3 s.h.
4. A course in geometrical structures; affine plane and space, Euclidean plane and space, trasformations, vectors. 3 s.h.
5. A first course in probability and statistics. 3 s.h.
6. A one year course in calculus of the real variable. 6 s.h.

7. A course in computers including some numerical analysis. 3 s.h.
8. A course in applications (uses of mathematics, selected from physics, astronomy, chemistry, biology, economics, sociology, business, linear programming, et. al.). 3 s.h.
9. A survey of junior high school mathematics from an advanced viewpoint (this is really junior high school mathematics professionalized for teachers). 3 s.h.
10. Teaching junior high school mathematics; concepts, methods, materials. 3 s.h.

In-service teacher training programs, conducted within a school system, have been highly successful in providing junior high school teachers with the mathematical background and experiences needed to teach SSMCIS courses 1 - 3. Often taught by experienced SSMCIS teachers, in-service programs have usually included a detailed coverage of a particular SSMCIS course, discussions of teaching strategies, lesson planning, and in some cases, actual "practice teaching" of SSMCIS classes for a few days.

Background of Courses 4 - 6.

To teach the senior high school program, additional background will be necessary. Since calculus in our program is taught from a contemporary viewpoint starting with continuity and using the topology of the line and plane, the study of calculus (or real analysis including differential equations) should be extended through another 6 s.h. For the teaching of probability another half year of study of probability and mathematical statistics (3 s.h.) would be required. A second course in vector spaces and linear algebra, introductory study of the complex variable, and further study in geometry would also be desirable. This additional study of approximately 15 semester hours can be obtained by increasing the number of hours of undergraduate study or by pursuing a master's level study as is now required by many states for certification.

Many teachers who have not had some of these courses can obtain a sufficient understanding of them by in-service study, either individually or in courses conducted by, or for, a group of teachers. In fact, it is this self-study that keeps one a scholar and alive in his field and it is to be recommended. It can also be achieved in summer institues where the emphasis is placed on putting all the worthwhile classical mathematics (that before 1900) and a very little of new mathematics into a contemporary setting. It is the way of conceiving of mathematics that is most of the new, modern, or contemporary in our subject at the secondary level of instruction.

While the "savoir" is important, even more important for the success of the teaching of SSMCIS is the "savoir faire." SSMCIS stresses the learning of concepts, understandings and problem solving as much (if not more so) as developing computional and manipulative skills, although the latter are not disregarded. The learning of mathematics calls for dynamic intellectual participation by the teacher and students. This comes about by inquiry into a problematic situation, proposing hypotheses, having active dialogue and debate--student with student--and students with teacher. By motivation through relevant and attractive situations we expect the student to raise issues and direct his own learning under teacher guidance, rather than having the teacher make all the moves on what to do. A home assignment should not be "do problems numbered so and so" but rather, "read and try to comprehend."

This way of learning can make education a joy and satisfaction for both the students and the teacher.

OFFICE FOR COMMUNICATION

The SSMCIS project will maintain an office, under the support of the National Science Foundation, for informational purposes from July 1, 1973 through June 30, 1974.

Persons desiring information or assistance may either write or phone the project office.

Mailing address: SSMCIS, Box 120
Teachers College, Columbia University
New York, N.Y. 10027

Telephone: (212) 870-4826

AVAILABILITY OF MATERIALS

A. To obtain textbooks and teachers commentaries please direct all orders to

Teachers College Press
Teachers College, Columbia University
1234 Amsterdam Avenue
New York, N.Y. 10027

UNIFIED MODERN MATHEMATICS

Course 1: Parts 1, 2 -- $3.25 per part

Course 1: Teachers Commentary -- $5.25

Course 2: Parts 1, 2 -- $3.25 per part

Course 2: Teachers Commentary -- $5.25

Course 3: Parts 1, 2 -- $3.25 per part

Course 3: Teachers Commentary -- $5.25

Course 4: Parts 1, 2 -- $2.50 per part

Course 4: Teachers Commentary -- $5.00

Course 5: Parts 1, 2 -- $2.50 per part

Course 5: Teachers Commentary -- $5.00

Course 6: 1 part only -- $3.00

Course 6: Teachers Commentary -- $5.00

Course 6 Booklets: Each Booklet -- $1.50

Booklet A -- Introduction to Statistical Inference

Booklet B -- Determinants, Matrices, and Eigenvalues

Booklet C -- Algebra Structures, Extensions, and Homomorphisms

Booklet D -- An Introduction to Differential Equations

Booklet E -- Geometric Mappings and Transformations

Course 6 Booklets: Teachers Commentary -- $5.00
(Booklets A through E combined)

B. The project offers the following informational materials which can be obtained without charge. Materials will be made available as long as supplies last.

I. Bulletin 6, An SSMCIS Report: MATHEMATICS EDUCATION IN EUROPE AND JAPAN

II. Information Bulletin 7

III. Technical Report Number 4: Performance of SSMCIS Students on the College Entrance Examination Board Mathematics Tests, Spring and Fall, 1971

IV. Additional reports on the following topics will become available during the 1973-1974 academic year

A. The Cumulative Progress Test: Courses 1-3

B. A manual for the Cumulative Progress Test

C. An investigation of the change in attitudes of SSMCIS students towards mathematics

D. A comparison of SSMCIS and non-SSMCIS student attitudes toward mathematics

E. A comparison of SSMCIS and non-SSMCIS student performance on the PSAT Examination.

Please direct all requests to:

SSMCIS, Box 120
Teachers College, Columbia University
New York, N.Y. 10027

COMMERCIAL PUBLICATION

Courses 4, 5, and 6 of the SSMCIS program are now being offered to commercial publishers of secondary school mathematics materials for consideration and bids for publishing rights. Until such commercial publications appear, probably by Fall, 1974, SSMCIS material may be purchased from Teachers College Press.

Courses 1, 2, and 3 were produced under a grant from the United States Office of Education and have been placed in the public domain without copyright privileges.

NOTICE

Due to an increase
in printing cost, the
price of each of the
Booklets A - E will be
$2.00
on all orders received
after July 1st, 1973.